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Neutrino mixing in a left-right model

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Abstract. We study the mixing among different generations of massive neutrino fields in a $SU(2)_L \times SU(2)_R \times U(1)_Y$ gauge theory which includes Majorana and Dirac mass terms in the Yukawa sector. Parity can be spontaneously broken at a scale $v_R \simeq 10^3 - 10^4$ GeV. We discuss about possible candidates for the Yukawa coupling matrices and we found that the model can accommodate a consistent pattern for neutral fermion masses as well as neutrino oscillations. The left and right sectors can be connected by a new neutral current.

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1 Introduction

The increasing experimental evidence on neutrino oscillations and non-zero masses [1] brings new light in some deep physical questions. The present value for the neutrino masses is consistent with a see-saw mass generation description involving a large mass scale. This suggests that grand unified theories have an important role in the neutrino mass spectrum. If this is the case, we still have many other points to clarify since the standard model has a relatively large number of input parameters and properties. In a recent work [2, 3], an extended model was proposed in other to clarify two of these points; the origin of parity breaking and the small value of the charged lepton mass spectrum relative to GUTs scales.

One possible way to understand the left-right asymmetry in weak interactions is to enlarge the standard model into a left-right symmetric structure and then, by some spontaneously broken mechanism, to recover the low energy asymmetric world. Many models were developed, based on grand unified groups [4], superstring inspired models [5], a connection between parity and the strong CP problem [6], left-right extended standard models [7]. All these approaches imply the existence of some new intermediate physical mass scale, well bellow the unification or the Planck mass scale. Left-right models starting from the gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ were developed by many authors [8] and are well known to be consistent with the standard $SU(2)_L \otimes U(1)_Y$. However, for the fermion mass spectrum there is no unique choice of the Higgs sector that can reproduce the observed values for both charged and neutral fermions, neither the fundamental fermionic representation is uniquely defined.

In [2], a left-right model with mirror fermions and a particular choice for the Higgs sector was proposed, leading to a new see-saw mass relation for both neutral and charged leptons. In the present paper we extend the model to three families and study the consequences for neutrino masses and oscillations. In Sect. 2 we briefly review the model for completeness. In Sects. 3 and 4 we present the charged and neutral lepton mass spectrum respectively, obtained from possible candidates for the Yukawa coupling matrices. In Sect. 5 we discuss, from the neutrino mixing matrix, the consequences for neutrino oscillations; in Sect. 6 we present the main phenomenological consequences and, finally, our conclusions are given in the last section.

2 The model

In this section we will briefly revise the most important features of the model. Details can be found in [2] and [3].

We will analyze the neutral fermion masses within the framework of a theory based on the gauge structure $SU(2)_L \times SU(2)_R \times U(1)_Y$ with coupling constants g_L,g_R and g.

An explicit realization of the model is provided by specifying its Higgs particle and fermion content. The first family assignment of standard and exotic fermions to the $SU(2)_L \times SU(2)_R \times U(1)_Y$ representations is as follows:

$$\begin{array}{llll} l_L = \left(\begin{array}{c} \nu \\ e \end{array} \right)_L & (1/2, 0, -1), & L_R = \left(\begin{array}{c} N \\ E \end{array} \right)_R & (0, 1/2, -1), \\ \quad \nu_R & (0, 0, 0), & \quad N_L & (0, 0, 0), \\ \quad e_R & (0, 0, -2), & \quad E_L & (0, 0, -2). \end{array} \tag{1}$$

The electric charge operator is defined in terms of the generators T_L, T_R and Y of $SU(2)_L$, $SU(2)_R$ and $U(1)$ respectively.

$$Q = (T_{3L} + T_{3R} + Y) \tag{2}$$

A general choice of the Higgs sector includes a Higgs field Φ in the mixed representation $(1/2, 1/2, 0)$ and two Higgs doublets

$$\chi_L = \left(\begin{array}{c} \chi_L^+ \\ \chi_L^0 \end{array} \right), \quad \chi_R = \left(\begin{array}{c} \chi_R^+ \\ \chi_R^0 \end{array} \right), \tag{3}$$

with transformation properties

$$(1/2, 0, 1)_{\chi_L}, \qquad (0, 1/2, 1)_{\chi_R}. \tag{4}$$

The breakdown of $SU(2)_L \times SU(2)_R \times U(1)_Y$ down to $U_{em}(1)$ is realized through a non-trivial pattern of vacuum expectations values for the Higgs fields, namely,

$$< \chi_L >= \left(\begin{array}{c} 0 \\ v_L \end{array} \right), \quad < \chi_R >= \left(\begin{array}{c} 0 \\ v_R \end{array} \right), \quad < \Phi >= \left(\begin{array}{cc} k & 0 \\ 0 & k' \end{array} \right). \tag{5}$$

Higgs doublets are responsible for the gauge boson and fermion masses. To the Higgs sector we add two new Higgs singlets, one coupled to Dirac mass terms – S_D – and the other coupled to Majorana mass terms – S_M.

Earlier left-right symmetric models [8] were built on a different basis. The fermionic content of the standard model is enlarged with new right handed doublets such as $(N, E)_R$. No new charged leptons are present as in our model. In the Higgs sector a fundamental role is given to the left-right mixed Higgs field Φ. In our model this field is not spontaneously broken as we will switch $<\Phi>= 0$ (i.e. $k = k' = 0$). As a consequence we have no charged vector boson $(W_L; W_R)$ mixing.

At present there are several indications in favor of non-zero neutrino mass and mixing between families coming from solar and atmospheric neutrino data [9]. Neutrinos are predicted to be Majorana particles in many extensions of the standard model containing neutrinos with non-zero masses. Here we will do so and allow Majorana mass terms within the Yukawa sector of the lagrangian. For the first family we have

$$\begin{aligned} \mathcal{L}_M &= f\left[\bar{l}_L\chi_L e_R + \overline{L}_R\chi_R E_L + \bar{l}_L\tilde{\chi}_L\nu_R + \overline{L}_R\tilde{\chi}_R N_L\right] + \\ &+ f'\left[\bar{l}_L\tilde{\chi}_L N_L^c + \overline{L}_R\tilde{\chi}_R\nu_R^c\right] + f''\left[\bar{l}_L\phi L_R\right] + \\ &+ gS_M\left[\overline{N^c}_L N_L + \overline{\nu^c}_R\nu_R\right] + g'S_D\overline{\nu}_R N_L + g''S_D\overline{e}_R E_L. \end{aligned} \tag{6}$$

The generalization to $n = 3$ families is straightforward. Notice that the inclusion of Majorana terms spoils the invariance with respect to any global gauge transformation so that there is no conserved leptonic charge (see for example [10]).

Fermions masses arise after spontaneous symmetry breaking of the gauge structure $SU(2)_L \times SU(2)_R \times U(1)_Y$ down to $SU(2)_L \times U(1)$. For the charged and neutral sectors the mass lagrangians are, respectively

$$\mathcal{L}_{M,c} = f\left[v_L\overline{e}_L e_R + v_R\overline{E}_R E_L\right] + g''s_D\overline{e}_R E_L + f''k'\overline{e}_L E_R + H.C., \tag{7}$$

and,

$$\begin{aligned} \mathcal{L}_{M,n} &= f\left[v_L\overline{\nu}_L\nu_R + v_R\overline{N}_R N_L\right] + f'\left[v_R\overline{N}_R\nu_R^c + v_L\overline{\nu}_L N_L^c\right] + f''k\overline{\nu}_L N_R \\ &+ gs_M\left[\overline{N^c}_L N_L + \overline{\nu^c}_R\nu_R\right] + g's_D\overline{\nu}_R N_L + H.C.. \end{aligned} \tag{8}$$

One of the main points of the mirror left-right model is the presence of the term $g''s_D\overline{e}_R E_L$ in the charged lepton mass matrix. This term will imply a see-saw mass relation for the charged sector. We have then a natural mechanism to explain small charged lepton masses in a large unified mass scale.

In this model CP violation is not considered and therefore the couplings f, f', g, g', g'' and h' are 3×3 real matrices.

In matrix form, taking $k = k' = 0$, the charged sector reads

$$\begin{aligned} \mathcal{L}_{M,c} &= \overline{\psi}M_c\psi, \\ &= (\overline{f}_L, \overline{F}_R, \overline{F}_L, \overline{f}_R)\begin{pmatrix} 0 & 0 & 0 & fv_L \\ 0 & 0 & fv_R & 0 \\ 0 & fv_R & 0 & g''s_D \\ fv_L & 0 & g''s_D & 0 \end{pmatrix}\begin{pmatrix} f_L \\ F_R \\ F_L \\ f_R \end{pmatrix}. \end{aligned} \tag{9}$$

On the other hand, for the neutral sector it is convienient to introduce the self-conjugated fields defined, for each family, as

$$\begin{aligned} \chi_\nu &= \nu_L + \nu_L^c \\ w_N &= N_R + N_R^c \\ \chi_N &= N_L + N_L^c \\ w_\nu &= \nu_R + \nu_R^c \end{aligned} \tag{10}$$

In terms of the new fields, equations (8) may be rewritten as

$$\begin{aligned} \mathcal{L}_{M,n} &= \overline{\xi} M_n \xi \\ &= (\overline{\chi}_\nu, \overline{w}_N, \overline{\chi}_N, \overline{w}_\nu) \begin{pmatrix} 0 & 0 & f' v_L & f v_L/2 \\ 0 & 0 & f v_R/2 & f' v_R \\ f' v_L & f v_R/2 & g s_M & g' s_D/2 \\ f v_L/2 & f' v_R & g' s_D/2 & g s_M \end{pmatrix} \begin{pmatrix} \chi_\nu \\ w_N \\ \chi_N \\ w_\nu \end{pmatrix}. \end{aligned} \tag{11}$$

The mass matrices show the following block structure with different mass scales

$$M = \begin{pmatrix} 0 & M_{LR} \\ M_{LR}^t & M_S \end{pmatrix} \tag{12}$$

where M_{LR} and M_S are $2n \times 2n$ matrices verifying $det(M_{LR}) \ll det(M_S)$.

In view of the see saw structure ($det(M_{LR}) \ll det(M_S)$), mass matrices can be driven to a block diagonal form by expanding in power series of $M_{LR} M_S^{-1}$ [11]. This results in a $2n \times 2n$ light fermion mass matrix and $2n \times 2n$ heavy one given by

$$M^{(light)} \simeq -M_{LR}^t M_S^{-1} M_{LR}, \qquad M^{(heavy)} = M_S \tag{13}$$

respectively.

3 Charged fermion masses

In order to obtain the fermion masses we need explicit textures for the coupling matrices in Eqs. (9) and (11). Mixing between families in the charged sector is phenomenologically disfavored and thus the coupling matrices f and g'' can be chosen diagonal. Taking $f = diag(1,1,1)$ and $g'' = diag(\lambda_1, \lambda_2, \lambda_3)$ we obtain for each family a light charged fermion, with mass eigenvalue $m_i = v_L v_R / \lambda_i s_D$, and a heavy one with eigenvalue $M_i = \lambda_i s_D$.

Flavor left handed and right handed fields ψ are connected to the physical fields η by means of an orthogonal transformation, that is

$$\psi_j = \sum_{k=1}^{4n} V_{jk} \eta_k \tag{14}$$

where η is the column matrix formed by the mass fields and n the number of families into consideration.

Explicitly, the mixing matrix V in the one family case is

$$V = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix} \tag{15}$$

From this matrix we can recover the Dirac structure of charged leptons by suitable rotations. The generalization to the three family case is easily done.

The three parameters λ_i in g'' allow us to recover the standard charged fermion spectrum in a simple way.

For the light fermions we have

$$m_i = \frac{v_L v_R}{\lambda_i s_D} \tag{16}$$

Fixing the vacuum parameter v_L equal to the Fermi scale v_{Fermi}, we obtain the following constraints

$$v_R/\lambda_1 s_D \simeq 10^{-6}, \qquad v_R/\lambda_2 s_D \simeq 10^{-3}, \qquad v_R/\lambda_3 s_D \simeq 10^{-2}. \tag{17}$$

Consequently, for $v_R \simeq 10^3 - 10^4\ GeV$ we have the following spectrum for the heavy sector

$$\begin{gathered} M_1 = \lambda_1 s_D = 10^9 - 10^{10}\ GeV, \quad M_2 = \lambda_2 s_D = 10^6 - 10^7\ GeV, \\ M_3 = \lambda_3 s_D = 10^5 - 10^6 GeV. \end{gathered} \tag{18}$$

Taking the vacuum expectation value of the Higgs scalar s_D at the mass scale 10^{10} GeV, then the coupling parameters λ_i are fixed to

$$\lambda_1 = 1, \qquad \lambda_2 = 10^{-3}, \qquad \lambda_3 = 10^{-4}. \tag{19}$$

4 Neutral fermion masses

The mass lagrangian corresponding to the neutral sector contains Dirac and Majorana mass terms built up from the inclusion of right handed neutrino fields and their mirror partners. In this framework, the description on the phenomenological neutrino mass matrix will differ from the most familiar schemes on three neutrino mixing found in the literature [12]. In equation (11), the Dirac mass terms arise from the off-diagonal submatrices of the blocks M_{LR} and M_S, while the Majorana terms arise from the diagonal ones. As we mention before, the difference between M_{LR} and M_S mass scales ensures the see-saw mechanism for the neutral sector. We still have to choose suitable candidates for the textures of the coupling matrices. There are many possibilities that can be compatible with the present experimental status on neutrino masses and oscillations.

For the Dirac mass terms we chose diagonal couplings. The simplest choice is to take f and g' equal to the unity.

An important point in the left-right symmetric model comes from the Majorana mass terms. Since Majorana fields are completely neutral and therefore, are all physically equivalent, it is a natural requirement that all Yukawa couplings are to be taken equal. This corresponds to taking democratic texture for the coupling f', that is

$$f' = \rho \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \tag{20}$$

where ρ may be set equal to 1 for simplicity. The analytic expression for $M^{(light)}$ is now

$$M^{(light)} = \frac{v_R^2}{s_M} \begin{pmatrix} \frac{1}{4}+3w^2 & 3w^2 & 3w^2 & \frac{1}{2}+\frac{1}{2}w^2 & \frac{1}{2}+\frac{1}{2}w^2 & \frac{1}{2}+\frac{1}{2}w^2 \\ 3w^2 & \frac{1}{4}+3w^2 & 3w^2 & \frac{1}{2}+\frac{1}{2}w^2 & \frac{1}{2}+\frac{1}{2}w^2 & \frac{1}{2}+\frac{1}{2}w^2 \\ 3w^2 & 3w^2 & \frac{1}{4}+3w^2 & \frac{1}{2}+\frac{1}{2}w^2 & \frac{1}{2}+\frac{1}{2}w^2 & \frac{1}{2}+\frac{1}{2}w^2 \\ \frac{1}{2}+\frac{1}{2}w^2 & \frac{1}{2}+\frac{1}{2}w^2 & \frac{1}{2}+\frac{1}{2}w^2 & 3+\frac{1}{4}w^2 & 3 & 3 \\ \frac{1}{2}+\frac{1}{2}w^2 & \frac{1}{2}+\frac{1}{2}w^2 & \frac{1}{2}+\frac{1}{2}w^2 & 3 & 3+\frac{1}{4}w^2 & 3 \\ \frac{1}{2}+\frac{1}{2}w^2 & \frac{1}{2}+\frac{1}{2}w^2 & \frac{1}{2}+\frac{1}{2}w^2 & 3 & 3 & 3+\frac{1}{4}w^2 \end{pmatrix} \tag{21}$$

where w is defined as $w \equiv v_L/v_R$.

In terms of the block matrices given in (11), $M^{(light)}$ reads

$$M^{(light)} = \frac{1}{s_M} \begin{pmatrix} f'^2 v_L^2 + f^2 \frac{v_R^2}{4} & \frac{f'f}{2}(v_L^2+v_R^2) \\ \frac{f'f}{2}(v_L^2+v_R^2) & f^2\frac{v_L^2}{4} + f'^2 v_R^2 \end{pmatrix} \tag{22}$$

In the last expression, the diagonal block matrices are decomposed into democratic matrices, with caracteristic mass scales v_L^2/s_M and v_R^2/s_M, respectively, and additional diagonal matrices coming from the Dirac mass terms of the Lagrangian. It is important to notice that, the underlying democratic symmetry constrains strongly the structure of the mixing matrix U and is, indeed, responsible for large mixing angle at both solar and atmospheric mass scales because it constraints the matrix elements $|U_{e6}|^2$, $|U_{\mu 6}|^2$ and $|U_{\tau 6}|^2$ to the values 0,1/2 and 1/2 respectively. This result had been used, previously, in models dealing with a 3 family neutrino mass matrix [13]. The extended $SU(2)_L \times SU(2)_R$ symmetry of the present model incorporates naturally two mass scales that will prove to be appropiate to accomodate both atmospheric and solar neutrino oscillations.

The interaction fields $\xi \equiv (\chi_\nu, w_N)^t$ are related to the physical ones $\eta \equiv (\nu, N)^t$ by means of the orthogonal transformation $\xi = U\eta$,

$$\begin{pmatrix} \chi_{\nu i} \\ w_{Ni} \end{pmatrix} = U \begin{pmatrix} \nu_i \\ N_i \end{pmatrix} \qquad i = 1, 2, 3 \tag{23}$$

The $2n \times 2n$ orthogonal matrix U is determined, in the limit where $det(M_{LR}) << det(M_S)$, by requiring

$$U^t M^{(light)} U = diag(m_1, m_2, \dots, m_6) \tag{24}$$

where m_k are the eigenvalues of $M^{(light)}$ and correspond to the spectrum of the light sector. Explicitly, we found

$$U = \begin{pmatrix} \sqrt{\frac{12}{37}} + O(w^2) & 0 & 0 & \sqrt{\frac{1}{111}} + O(w^2) & -\sqrt{\frac{2}{3}} & 0 \\ \sqrt{\frac{12}{37}} + O(w^2) & 0 & 0 & \sqrt{\frac{1}{111}} + O(w^2) & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ \sqrt{\frac{12}{37}} + O(w^2) & 0 & 0 & \sqrt{\frac{1}{111}} + O(w^2) & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \\ -\sqrt{\frac{1}{111}} + O(w^2) & -\sqrt{\frac{2}{3}} & 0 & \sqrt{\frac{12}{37}} + O(w^2) & 0 & 0 \\ -\sqrt{\frac{1}{111}} + O(w^2) & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \sqrt{\frac{12}{37}} + O(w^2) & 0 & 0 \\ -\sqrt{\frac{1}{111}} + O(w^2) & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \sqrt{\frac{12}{37}} + 0(w^2) & 0 & 0 \end{pmatrix} \tag{25}$$

The neutrino fields are labeled ν or N according to their characteristic mass scales v_L^2/s_M or v_R^2/s_M, respectively. The spectrum of light Majorana neutrino masses is

$$\begin{aligned} m_{\nu_1} &= \frac{1}{4}\frac{v_L^2}{s_M}, \qquad m_{\nu_2} = \frac{1}{4}\frac{v_L^2}{s_M}, \qquad m_{\nu_3} \simeq \frac{1225}{148}\frac{v_L^2}{s_M} \\ m_{N_1} &= \frac{1}{4}\frac{v_R^2}{s_M}, \qquad m_{N_2} = \frac{1}{4}\frac{v_R^2}{s_M}, \qquad m_{N_3} \simeq \frac{37}{4}\frac{v_R^2}{s_M}. \end{aligned} \tag{26}$$

It is interesting to notice that the model leads naturally to a hierarchical mass spectrum, with different square mass scales. As we will see in the next section, this feature is essential to account for the mass pattern coming from neutrino oscillation data. From Eq. (26) we can also redefine the six Majorana fields in terms of two Dirac and two Majorana neutrino fields.

The main theoretical constraints on neutrino masses come from cosmological considerations related to typical bounds on the universe mass density and its lifetime. Specifically, the cosmological bound follows from avoiding the overabundance of relic neutrinos. For neutrinos below $\simeq 1\ MeV$ the limit on masses for Majorana type neutrinos is [14]

$$\sum_\nu m_\nu \leq 100\Omega_\nu h^2\ eV \simeq 30\ eV \tag{27}$$

where Ω_ν is the neutrino contribution to the cosmological density parameter, Ω, defined as the ratio of the total matter density to the critical energy density of the universe and the factor h^2 measures the uncertainty in the determination of the present value Hubble parameter h. The factor Ωh^2 is known to be smaller than 1.

In Eq. (27) the matter component represented by the factor $\Omega_\nu h^2$ was chosen smaller than 0.3, according to reference [15] , in order to obtain an age of the Universe $t \geq 12$ Gyears.

From (26), the sum of neutrino masses satisfy

$$\sum_i^6 m_i \lesssim 10\frac{v_R^2}{s_M}, \tag{28}$$

so that the cosmological criterium (27) is verified if

$$\frac{v_R^2}{s_M} \lesssim 10\Omega_\nu h^2 \; eV. \tag{29}$$

This constrains the breaking scale s_M to be $s_M \gtrsim 10^{15} \; GeV$ when v_R is fixed at $v_R \simeq 10^3 \; GeV$.

5 Oscillations of neutrinos

The oscillations in neutrino beams are one of the most fundamental consequences of neutrino mixing. Experimental results concerning a two-generation transition are quoted in terms of $\Delta m^2 = m_2^2 - m_1^2$, and the mixing angle. We will see in this section that the model presented previously yields satisfactory results for the democratic texture of the Majorana terms coupling matrices when we fix $v_R^2/s_M \simeq 10^{-2} eV$.

Taking into account the orthogonality of the mixing matrix, the probability of transition $\nu_\alpha \to \nu_\beta$ between two generations α and β is

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta) &= |\sum_i^{2n} U_{\alpha i} U_{\beta i} \exp(-i\Delta m_{i1}^2 L/2E)|^2 \\ &= |\delta_{\alpha\beta} + \sum_i^{2n} U_{\alpha i} U_{\beta i} \left[\exp(-i\Delta m_{i1}^2 L/2E) - 1\right]|^2 \end{aligned} \tag{30}$$

where $L \simeq t$ is the distance between neutrino source and neutrino detector and E is the neutrino energy.

Notice that as a general feature of the transition probability, neutrino oscillations can be observed whenever the condition $\Delta m_{i1} L/E \sim 1$ is satisfied.

Specifically, considering the model in question supplemented by the democratic texture input, we obtain for the $\nu_e \to \nu_\mu$ transition

$$\begin{aligned} P(\nu_e \to \nu_\mu) &= |U_{e4} U_{\mu 4} \left[\exp(-i\Delta m_{41}^2 L/2E) - 1\right] + \\ &\quad + U_{e5} U_{\mu 5} \left[\exp(-i\Delta m_{51}^2 L/2E) - 1\right]|^2, \end{aligned} \tag{31}$$

where the explicit values of the matrix elements U_{ei} are given in Eq. (25).

In first approximation we neglect $|U_{e4}U_{\mu 4}| = 1/111$ in front of $|U_{e5}U_{\mu 5}| = 1/3$, yielding to the simpler expression

$$\begin{aligned} P(\nu_e \to \nu_\mu) &\simeq |U_{e5} U_{\mu 5} \left[\exp(-i\Delta m_{51}^2 L/2E) - 1\right]|^2 \\ &\simeq \frac{1}{2} 4|U_{e5}|^2 |U_{\mu 5}|^2 \left(1 - \cos \Delta m_{51}^2 \frac{L}{2E}\right) \end{aligned} \tag{32}$$

and therefore the amplitude of the probability mixing, $P(\nu_e \to \nu_\mu)$ and the relevant scale of mass are

$$4|U_{e5}|^2 |U_{\mu 5}|^2 = \frac{4}{9}, \qquad \Delta m^2 = \Delta m_{51}^2 \simeq \frac{1}{16} \frac{v_R^4}{s_M^2}. \tag{33}$$

An analogous calculation yields the same result for the $\nu_e \to \nu_\tau$ transition. The reader should notice that the expression for the transition probability is similar to that obtained for the 2-neutrino mixing case, however the present model predicts two domintant ν_e transitions.

Recent solar neutrino oscillations results (SNO) strongly favor the large mixing angle Mikheyev-Smirnov-Wolfenstein (MSW) solar solution at the scale

$$\Delta m^2_{sol} \simeq 10^{-5} eV^2 \tag{34}$$

Replacing this value in Eq. (33) and choosing $v_R \simeq 10^3 \; GeV$, we found that in the left-right-mirror model the singlet breaking scale should be fixed $s_M \simeq 10^{17} \; GeV$ in order to recover the mass domain of the solar neutrino experimental results.

According to Eq. (30), the survival probability $P(\nu_e \to \nu_e)$ is given by

$$\begin{aligned} P(\nu_e \to \nu_e) &= |U_{e1}U_{e1} + U_{e5}U_{e5} \exp(-i\Delta m^2_{51} L/2E)|^2 \\ &= |U_{e1}|^4 + |U_{e5}|^4 + 2|U_{e1}|^2|U_{e5}|^2 \cos \Delta m^2_{51} \frac{L}{2E} \end{aligned} \tag{35}$$

which can be rewritten, using the orthogonality property of the mixing matrix and neglecting $|U_{e4}|^2 \simeq 0$, as

$$P(\nu_e \to \nu_e) = 1 - 2|U_{e1}|^2|U_{e5}|^2 \left(1 - \cos \Delta m^2_{51} \frac{L}{2E}\right) \tag{36}$$

This result can be directly compared with the average survival probability, $P^{3\nu}(\nu_e \to \nu_e)$, obtained in the framework of three neutrino mixing supplemented with the mass hierarchy condition

$$\Delta m^2_{21} \simeq \Delta m^2_{sol}; \; \Delta m^2_{32} \simeq \Delta m^2_{atm}; \; \Delta m^2_{21} << \Delta m^2_{32} \tag{37}$$

Due to the hierarchy, and considering that in the three neutrino mixing scenarios the experimental data constrains the mixing angle θ_{13} to be small (see for example reports of the CHOOZ collaboration [16]), the survival probability $P^{3\nu}$ is completely determined by two parameters as in the 2 neutrino mixing case, and is given by the following expression

$$P^{3\nu}(\nu_e \to \nu_e) = 1 - \frac{1}{2} \sin^2 2\theta_{sol} \left(1 - \cos \Delta m^2_{sol} \frac{L}{2E}\right). \tag{38}$$

The amplitude of the oscillating term in Eq. (36) predicted by the present model, when compared to Eq. (38) gives

$$\sin^2 2\theta \equiv 4|U_{e1}|^2|U_{e5}|^2 = \frac{32}{37}, \text{ or } \tan^2 \theta \simeq 0.46, \tag{39}$$

where we used the orthogonality of the mixing matrix U to reduce the parameter dependence of the amplitude to a single angle, θ.

The above result lies inside the allowed experimental mixing angle region of the solar solution, namely [16, 17],

$$\tan^2 \theta_{sol} = 0.35 \pm^{0.3}_{0.1}. \tag{40}$$

It is important to notice that $U_{e6} = 0$ guarantees large mixing at the solar mass scale and plays the same role as V_{e3} does in the framework of a 3 neutrino mixing described by a 3×3 mixing matrix V. The null value of U_{e6} comes from the underlying democratic symmetry of the diagonal blocks in Eq. (22).

We now turn our attention to the $\nu_\mu \to \nu_\tau$ transition. In this case, Eq. (30) leads to

$$\begin{aligned} P(\nu_\mu \to \nu_\tau) &= |U_{\mu 5} U_{\tau 5} \left[\exp(-i\Delta m^2_{51} L/2E) - 1\right] + \\ &\quad + U_{\mu 6} U_{\tau 6} \left[\exp(-i\Delta m^2_{61} L/2E) - 1\right] |^2 \end{aligned} \tag{41}$$

Now the oscillations are also characterized by a new scale of masses, namely Δm^2_{61} that didn't appear in the transitions $\nu_e \to \nu_\mu, \nu_\tau$. Using the estimate value for s_M, we found

$$\Delta m^2_{61} \simeq \left(\frac{37}{4}\right)^2 \frac{v_R^4}{s_M^2} \simeq 10^{-3}\ eV^2. \tag{42}$$

Defining de oscillations lenghts as

$$L_{61} = 2\pi \frac{2E}{\Delta m^2_{61}},\ ,\ L_{51} = 2\pi \frac{2E}{\Delta m^2_{51}}, \tag{43}$$

notice that for the relevant values L/E for neutrino oscillation in the atmospheric range (i.e. $\Delta m^2_{61} L/E \simeq 1$), we have $L/L_{51} << L/L_{61}$, and we can neglect the contribution of Δm^2_{51} to the transition probability (41). Thus Eq. (41) may be approximated by

$$\begin{aligned} P(\nu_\mu \to \nu_\tau) &= |U_{\mu 6} U_{\tau 6} \left[\exp(-i\Delta m^2_{61} L/2E) - 1\right] |^2 \qquad (44) \\ &= 2|U_{\mu 6}|^2 |U_{\tau 6}|^2 \cos \Delta m^2_{61} \frac{L}{2E} \end{aligned}$$

which implies large mixing at the scale Δm^2_{16}.

The recent data on atmospheric neutrino by Super-Kamiokande [17] show that the origin of the zenith angle dependence of the neutrino flux is due to oscillations between ν_μ and ν_τ. The data is consistent with maximal ν_μ and ν_τ mixing at a square mass difference scale $\Delta m^2_{atm} \simeq 10^{-3}\ eV^2$. Indeed, the preferable experimental values of mass and mixing parameters are

$$\sin^2 2\theta_{atm} = 1.0, \qquad \Delta m^2_{atm} = 3.5 \times 10^{-3}\ eV^2. \tag{45}$$

Those values are in agreement with the parameters, given in Eq. (44), predicted by the mirror model for the $\nu \to \tau$ transition, that is,

$$\sin^2 2\theta' = 1, \qquad \Delta m^2_{16} \simeq \left(\frac{37}{4}\right)^2 \frac{v_R^4}{s_M^2} \tag{46}$$

where the mixing angle θ' is defined, using the orthogonality property of U, through the relation $\sin^2 2\theta' = 4|U_{\mu 6}|^2 |U_{\tau 6}|^2$.

Summarizing, the extended right symmetry of the mirror model, broken at the v_R scale, provides the means to accomodate neutrino oscillations at the solar and atmospheric mass scales. Futhermore, when supplemented by an explicit democratic symmetry in the Majorana mass terms, the model leads to a hierarchical neutrino spectrum and a mixing matrix structure that assures the decoupling of atmospheric from solar neutrino oscillations, because it fixes $|U_{e6}| = 0$. The democratic coupling implies also large mixing angles for both solar and atmospheric problems.

As a last comment, we observe from the mixing matrix that the model predicts a similar pattern of oscillation for the mirror fields w_{Ni} but in the range of v_L^4/S_M^2 mass scales. The model also predicts transitions between stantard and exotic neutrinos. These, however, are rather suppressed by the small amplitude $\propto 7 \times 10^{-3}$, as can be deduced from Eq. (25).

6 Phenomenology

In order to analyze some phenomenological consequences of the model we'll work out the interaction Lagrangian. We will see that the standard model results are safely recovered at the Fermi scale and that the connection between the left and right sectors appears at the breaking scale of the new gauge group $SU(2)_R$ where non-negligible effects, involving a new neutral current, are predicted.

As done elsewhere [18], grouping all fermions of a given electric charge and a given helicity ($h = L, R$) in a vector column $\psi_h = (\psi_O, \psi_E)_h^t$ of n ordinary (O) and m exotic (E) gauge eingenstates, the interaction Lagrangian for the neutral current is simply written as

$$\mathcal{L}^{nc} = \sum_h \overline{\psi}_h \gamma^\mu \left(g_L T_L^3, g_R T_R^3, g\frac{Y}{2} \right) \psi_h \begin{pmatrix} W_L^3 \\ W_R^3 \\ B \end{pmatrix}, \tag{47}$$

or, in terms of the physical neutral vector bosons (Z, Z', A)

$$\mathcal{L}^{nc} = \sum_h \overline{\psi}_h \gamma^\mu R^t \left(g_L T_L^3, g_R T_R^3, g\frac{Y}{2} \right) \psi_h R \begin{pmatrix} Z \\ Z' \\ A \end{pmatrix}. \tag{48}$$

R is a 3×3 matrix representation of the orthogonal transformation which connects the weak $(W_{L\mu}^3, W_{R\mu}^3, B_\mu)$ and mass eigenstates basis (Z_μ, Z'_μ, A_μ). In its standard form,

$$R = \begin{pmatrix} c_{\theta_w} c_\alpha & c_{\theta_w} s_\alpha & s_{\theta_W} \\ -s_\alpha c_\beta - c_\alpha s_{\theta_W} s_\beta & c_\beta c_\alpha - s_\alpha s_{\theta_W} s_\beta & s_\beta c_{\theta_W} \\ s_\alpha s_\beta - c_\alpha s_{\theta_W} c_\beta & -s_\beta c_\alpha - s_\alpha s_{\theta_W} c_\beta & c_\beta c_{\theta_W} \end{pmatrix} \tag{49}$$

where θ_W, α and β are the mixing angles between the $Z - A$, $Z - Z'$ and $Z' - A$ bosons.

By direct calculation from the neutral bosons mass matrix one can obtain an analytic expression for R in powers of $w = v_L/v_R$

$$R = \begin{pmatrix} \frac{g_L(g_R^2+g^2)^{1/2}}{\Delta^{1/2}} + O(w^4) & \frac{g_L g^2}{(g_R^2+g^2)^{3/2}} w^2 & \frac{g_R g}{\Delta^{1/2}} \\ -\frac{g^2 g_R}{\Delta^{1/2}(g_R^2+g^2)^{1/2}} - \frac{g_R g^2 \Delta^{1/2}}{(g^2+g_R^2)^{5/2}} w^2 & \frac{g_R}{(g_R^2+g^2)^2} - \frac{g_R g^4}{(g_R^2+g^2)^{5/2}} w^2 & \frac{g_L g}{\Delta^{1/2}} \\ -\frac{g^2 g_R}{\Delta^{1/2}(g_R^2+g^2)^{1/2}} + \frac{g^3 \Delta^{1/2}}{(g^2+g_R^2)^{5/2}} w^2 & -\frac{g}{(g_R^2+g^2)^2} - \frac{g_R^2 g^3}{(g_R^2+g^2)^{5/2}} w^2 & \frac{g_R g_L}{\Delta^{1/2}} \end{pmatrix} \tag{50}$$

with $\Delta = g_L^2 g_R^2 + g_L^2 g^2 + g_R^2 g^2$.

In the limit $w = 0$, which corresponds to no mixing between $Z - Z'$ (or $\alpha = 0$), one recovers the standard model case.

The following identities arise by comparying Eqs. (49) and (50),

$$\sin^2\theta_W = \frac{g_R^2 g^2}{g_R^2 g_L^2 + g_R^2 g^2 + g_L^2 g^2}, \qquad \sin^2\beta = \frac{g^2}{g_R^2 + g^2}. \tag{51}$$

Expressed in terms of the rotation angles, the neutral currents in (47) coupled to the massive vector bosons Z and Z' are respectively,

$$J_\mu = \frac{g_L}{\cos\theta_W}\gamma_\mu \left[(1 - w^2\sin^4\beta)T_{3L} - w^2\sin^2\beta T_{3R} - Q\sin^2\theta_W(1 - w^2\frac{\sin^4\beta}{\sin^2\theta_W})\right] \tag{52}$$

$$J'_\mu = g_L\tan\theta_W\tan\beta\left[\left(1 + w^2\frac{\sin^2\beta\cos^2\beta}{\sin^2\theta_W}\right)T_{3L} + \frac{T_{3R}}{\sin^2\beta} - Q(1 + w^2\sin^2\beta\cos^2\beta)\right]. \tag{53}$$

The corrections to the standard model neutrino NC coming from the extended group symmetry are

$$\mathcal{L}^{\nu,\mathcal{N}} = -J_\mu^{\nu,N} Z^\mu - J'^{\nu,N}_\mu Z'^\mu \tag{54}$$

$$= -\frac{g_L}{2\cos\theta_W}\left[(1 - w^2\sin^4\beta)\overline{\nu}_L\gamma^\mu\nu_L - w^2\sin^2\beta\overline{N}_R\gamma^\mu N_R\right] Z_\mu$$
$$-\frac{1}{2}g_L\tan\theta_W\tan\beta\left[\left(1 + w^2\frac{\sin^2\beta\cos^2\beta}{\sin^2\theta_W}\right)\overline{\nu}_L\gamma^\mu\nu_L + \frac{1}{\sin^2\beta}\overline{N}_R\gamma^\mu N_R\right] Z'_\mu, \tag{55}$$

or, in terms of the Majorana fields defined in (10),

$$\mathcal{L}^{\nu,\mathcal{N}} = -\frac{g_L}{2\cos\theta_W}\left[(1 - w^2\sin^4\beta)\sum_i^n \overline{\chi_{\nu i}}\gamma_\mu\frac{(1-\gamma_5)}{2}\chi_{\nu i} + - w^2\sin^2\beta\sum_i^n \overline{w_{Ni}}\gamma_\mu\frac{(1+\gamma_5)}{2}w_{Ni}\right] Z^\mu$$
$$- \frac{1}{2}g_L\tan\theta_W\tan\beta\left[\left(1 + w^2\frac{\sin^2\beta\cos^2\beta}{\sin^2\theta_W}\right)\sum_i^n \overline{\chi_{\nu i}}\gamma_\mu\frac{(1-\gamma_5)}{2}\chi_{\nu i} + + \frac{1}{\sin^2\beta}\sum_i^n \overline{w_{Ni}}\gamma_\mu\frac{(1+\gamma_5)}{2}w_{Ni}\right] Z'^\mu \tag{56}$$

In order to express the neutral currents in terms of mass eigenstates one has to use the transformation relation (23) into the interaction lagrangian (54). This yields

$$\begin{aligned}
\mathcal{L} &= -J^{\nu,N}_{\mu} Z^{\mu} - J'^{\nu,N}_{\mu} Z'^{\mu} \qquad (57)\\
&= -\frac{g_L}{2\cos\theta_W}\Bigg[(1-w^2\sin^4\beta)\sum_{i=1}^{3}\sum_{jk}^{2n}(U_{ij}U_{ik})\overline{\eta}_j\gamma_\mu\frac{(1-\gamma_5)}{2}\eta_k +\\
&- w^2\sin^2\beta\sum_{i=4}^{6}\sum_{jk}^{2n}(U_{ij}U_{ik})\overline{\eta}_j\gamma_\mu\frac{(1+\gamma_5)}{2}\eta_k\Bigg] Z^{\mu}\\
&-\frac{1}{2}g_L\tan\theta_W\tan\beta\Bigg[\left(1+w^2\frac{\sin^2\beta\cos^2\beta}{\sin^2\theta_W}\right)\sum_{i=1}^{3}\sum_{jk}^{2n}(U_{ij}U_{ik})\overline{\eta}_j\gamma_\mu\frac{(1-\gamma_5)}{2}\eta_k +\\
&+ \frac{1}{\sin^2\beta}\sum_{i=4}^{6}\sum_{j,k}^{2n}(U_{ij}U_{ik})\overline{\eta}_j\gamma_\mu\frac{(1+\gamma_5)}{2}\eta_k\Bigg] Z'^{\mu}
\end{aligned}$$

As a consequence of the neutral gauge bosons mixing, mirror neutrinos couple to the Z boson and may contribute to Z decay Γ_Z. Correspondence with the experimental results may be achieved by constraining the angle α, or equivalently, the factor w^2, which parametrize the $Z - Z'$ mixing. This is indeed the case for $v_R > 30 v_L$ [2]. It should be noticed that the non-standard $E_L - Z$ coupling contains a term that is not suppressed by a w^2 factor, namely,

$$\mathcal{L}^{E_L}_{Z'} = -g_L\tan\theta_W\tan\beta\overline{E_L}\gamma_\mu E_L. \qquad (58)$$

However, this contribution is excluded at energies lying in the electroweak scale due to the large charged fermion masses in the heavy sector (see Sect. 3). Therefore, the standard model results are recovered in the limit $w^2 << 1$.

The neutral current coupled to the massive vector boson Z' contains non-suppressed couplings which involves either standard or exotic neutrinos and are important to test the model at the $SU(2)_R$ breaking scale.

These contributions are

$$\begin{aligned}
\mathcal{L}_{\mathcal{Z}'} &= -\frac{1}{2}g_L\tan\theta_W\tan\beta\Bigg[\sum_{i=1}^{3}\sum_{jk}^{2n}(U_{ij}U_{ik})\overline{\eta}_j\gamma_\mu\frac{(1-\gamma_5)}{2}\eta_k + \qquad (59)\\
&+ \frac{1}{\sin^2\beta}\sum_{i=4}^{6}\sum_{j,k}^{2n}(U_{ij}U_{ik})\overline{\eta}_j\gamma_\mu\frac{(1+\gamma_5)}{2}\eta_k\Bigg] Z'^{\mu}
\end{aligned}$$

The new Z' gauge boson can be produced at the Large Hadron Collider with masses in the 1-4 TeV region [2]. The implications of a new Z' to the high precision electroweak data was studied by Erler and Langacker [19].

7 Conclusion

The recent experimental reports on neutrino oscillations, suggesting non-zero masses for neutrinos, are certainly the strongest indication for physics beyond the standard model. Enlarging the fermion spectrum by introducing mirror matter is a simple way to implement non-zero neutrino masses in extended theories. In the present paper we saw that a consistent spectrum of neutrino masses and oscillation pattern can arise in a such a scenario, which is motivated by an underlying left-right symmetric structure in the gauge group. We show that the well known $SU(2)_L \times SU(2)_R \times U(1)_Y$ theory, spontaneously broken into the standard $SU(2)_L \times U(1)$ at the mass scale $v_R \simeq 10^3\ GeV$, and supplemented by two Higgs singlets with vacuum parameters at the scales $s_D \simeq 10^{10}\ GeV$ and $s_M \simeq 10^{17}\ GeV$ reproduce the observed charged and neutral fermion masses. Explicit use of democratic textures for the coupling matrix of the Majorana mass term leads to suitable mixing parameters for the solar and atmospheric neutrino oscillation problems.

The new physics predicted by the model is consistent with the theoretical arguments and experimental results available on neutrino physics. The connection between the known leptons and their mirror states can be experimentally tested by a new neutral gauge boson present at the TeV mass scale. New Majorana neutrinos, such as those considered here, may be experimentally tested at the large hadron collider at CERN [20].

The satisfactory results presented in this work motivates further extensions of the model including both CP violation effects in neutrino oscillations and the LSND experimental results, if confirmed. This points will imply modifications in the Yukawa couplings and the choice of the Higgs sector.

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